Quadratics 7.2/7.3 : The Quadratic Formula

Quad Formula : $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

① $\left(\frac{8}{3}m^2 - 2m + \frac{2}{5} = 0\right) \times 15$

$40m^2 - 30m + 6 = 0$

$m = \dfrac{30 \pm \sqrt{900 - 4(40)(6)}}{2(40)}$

$= \dfrac{30 \pm \sqrt{-60}}{80}$

$= \dfrac{30 \pm 2i\sqrt{15}}{80}$

$= \dfrac{3}{8} \pm \dfrac{\sqrt{15}}{40} i$

② $x^2 + 2ix = 1$

$x^2 + 2ix - 1 = 0$

$a = 1 \quad b = 2i \quad c = -1$

$x = \dfrac{-2i \pm \sqrt{(2i)^2 - 4(1)(-1)}}{2(1)}$

$= \dfrac{-2i \pm \sqrt{0}}{2} = -i$

TRY: $x^2 - 2x\sqrt{3} - 3 = 0$

$a = 1, \; b = -2\sqrt{3}, \; c = -3$

$x = \dfrac{2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(1)(-3)}}{2}$

$= \dfrac{2\sqrt{3} \pm \sqrt{24}}{2}$

$= \dfrac{2\sqrt{3} \pm 2\sqrt{6}}{2} = \sqrt{3} \pm \sqrt{6}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ — discriminant

b^2-4ac : if perfect $\square$, then factorable

if positive, 2 real roots

if zero, 1 real root

if negative, 2 imaginary conjugates

Ex) Determine the nature of the roots.

$\left(x^2+\frac{7}{3}x+\frac{2}{3}=0\right)\times 3$

$3x^2+7x+2=0$

$(7)^2-4(3)(2)$

$49-24=25$ factorable, 2 real roots

Find the values of k for which

$3x^2-2x+k=0$ has ...

① 1 real double root

$(-2)^2-4(3)(k)=0$

$4-12k=0$

$k=\frac{1}{3}$

② 2 different real roots

$(-2)^2-4(3)(k)>0$

$4-12k>0$

$\frac{-12k}{-12} > \frac{-4}{-12}$

$k<\frac{1}{3}$

$(-\infty, \frac{1}{3})$

③ 2 imaginary conjugates

$4-12k<0$

$\frac{4}{12} < \frac{12k}{12}$

$\frac{1}{3}<k$

$(\frac{1}{3}, \infty)$

① $2x^2-5x+7=0$

2 solutions:

$\frac{5+i\sqrt{31}}{4}$ and $\frac{5-i\sqrt{31}}{4}$

☆ sum of solutions

$\frac{5+i\sqrt{31}}{4} + \frac{5-i\sqrt{31}}{4} = \frac{10}{4} = \frac{5}{2}$

☆ product of solutions

$\left(\frac{5+i\sqrt{31}}{4}\right)\left(\frac{5-i\sqrt{31}}{4}\right) = \frac{36}{16} = \frac{7}{2}$

② $2x^2+13x+11=0$

2 solutions:

-1 and $-\frac{11}{2}$

☆ sum of solutions

$-1 + \frac{-11}{2} = \frac{-13}{2}$

☆ product of solutions

$(-1)\left(-\frac{11}{2}\right) = \frac{11}{2}$

① $2x^2-5x+7=0$

② $2x^2+13x+11=0$

Conclusion:

For $ax^2+bx+c=0$

Sum of roots: $\frac{-b}{a}$, product of roots: $\frac{c}{a}$

HW: A15 p314 # 7, 15, 17, 23, 33, 37

p 320 # 5-11 odds, 16-17, 24, 31-37 odd, 40